

## Prime and Composite Numbers

Natural numbers are the set of numbers we use when we count.

$\{1, 2, 3, 4, 5, 6, 7, \dots\}$

A *prime number* is a natural number that can be divided, without leaving any remainder, by only itself and one. A prime number has only two factors, itself and one.

For example, 5 can be divided, without a remainder, only by 5 and 1.

5 has exactly two natural number factors, 5 and 1.

5 is a prime number.

A *composite number* is an natural number that can be divided, without leaving any remainder, by a natural number other than itself and one.

For example,

$$2 = 1 \times 2$$

Prime

$$3 = 1 \times 3$$

Prime

$$6 = 1 \times 6 \text{ and } 2 \times 3$$

Composite

6 can be divided by 2 and by 3, so 6 is composite.

$$15 = 1 \times 15 \text{ and } 3 \times 5$$

Composite

15 can be divided by 3 and by 5, so 15 is composite.

Interesting fact 1:

There is exactly one even prime number. It is also the smallest prime number. Do you know what it is?

Interesting fact 2:

Prime numbers are interesting to scientists, especially the large ones. Large prime numbers are used as keys in the codes that are used to send secret messages. Since these are not easy to find, the codes are difficult to break.

When you try to decide if a larger number is prime, you really only need to find out if it is divisible by prime numbers that are less than it is.

## Prime Factorization

Every number has one prime factorization.

Example: Find the prime factorization of 159.

159 is not divisible by 2 or any other even number.

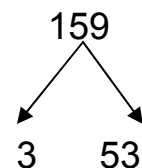
159 is divisible by 3:  $159 = 3 \times 53$

53 is a prime number.

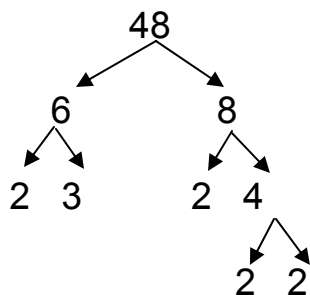
3 is a prime number.

The prime factorization of 159 is  $159 = 3 \times 53$ .

You could use a factor tree to find the prime factors:

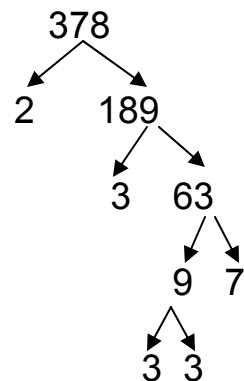


Factor trees may extend further:



The prime factorization of 48 is:

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$



The prime factorization of 378 is:

$$378 = 2 \times 3 \times 3 \times 3 \times 7$$

Now, find the prime factor of each of the following natural numbers, and then classify each as *prime* or *composite*:

1) 17

\_\_\_\_\_

2) 27

\_\_\_\_\_

3) 92

\_\_\_\_\_

4) 157

\_\_\_\_\_

5) 51

\_\_\_\_\_

6) 295

\_\_\_\_\_

7) 143

\_\_\_\_\_

## The Sieve of Eratosthenes

Over 2000 years ago, Eratosthenes, a Greek who studied mathematics and was the third librarian of the library at Alexandria, was also interested in prime numbers. He lived from 276 BC until 194 BC. He is credited with the development of what we now call the “Sieve of Eratosthenes”.

He arranged numbers in ten columns. One (1) is not a prime number. Its only factor is itself. Then, since 2 is the first prime number, he crossed off every second number following 2, like this:

1	2	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	9	<del>10</del>
11	<del>12</del>	13	<del>14</del>	15	<del>16</del>	17	<del>18</del>	19	<del>20</del>
21	<del>22</del>	23	<del>24</del>	25	<del>26</del>	27	<del>28</del>	29	<del>30</del>
31	<del>32</del>	33	<del>34</del>	35	<del>36</del>	37	<del>38</del>	39	<del>40</del>
41	<del>42</del>	43	<del>44</del>	45	<del>46</del>	47	<del>48</del>	49	<del>50</del>

3 is the next prime number. So Eratosthenes crossed out every third number.

1	2	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	<del>9</del>	<del>10</del>
11	<del>12</del>	13	<del>14</del>	<del>15</del>	<del>16</del>	17	<del>18</del>	19	<del>20</del>
<del>21</del>	<del>22</del>	23	<del>24</del>	25	<del>26</del>	<del>27</del>	<del>28</del>	29	<del>30</del>
31	<del>32</del>	<del>33</del>	<del>34</del>	35	<del>36</del>	37	<del>38</del>	<del>39</del>	<del>40</del>
41	<del>42</del>	43	<del>44</del>	<del>45</del>	<del>46</del>	47	<del>48</del>	49	<del>50</del>

The next number that has not been crossed out is the next prime number. This time it is 5. Eratosthenes circled the five and crossed out every fifth number.

This process can be continued as long as you have time, patience, and a list of counting numbers. All of the numbers that are crossed out are composite numbers.

On the next page is a list of numbers from 1 to 200. Use the sieve of Eratosthenes to find all of the prime numbers less than 200.

## The Sieve of Eratosthenes

Cross out 1. It is not a prime number.

Circle 2.

Cross out all of the multiples of 2: 4, 6, 8, 10, 12, 14, 16, ...

Circle the next number that has not been crossed out: 3.

Cross out the multiples of 3.

Continue until all of the numbers that are multiples of the primes have been crossed out. The numbers that are left are prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

## Answers

### Prime Factorization

1) 17	$1 \times 17$	Prime
2) 27	$3 \times 3 \times 3$	Composite
3) 92	$2 \times 2 \times 23$	Composite
4) 157	$1 \times 157$	Prime
5) 51	$3 \times 17$	Composite
6) 295	$5 \times 59$	Composite
7) 143	$11 \times 13$	Composite

### The Sieve of Eratosthenes

(the prime numbers are circled)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200